8th Grade

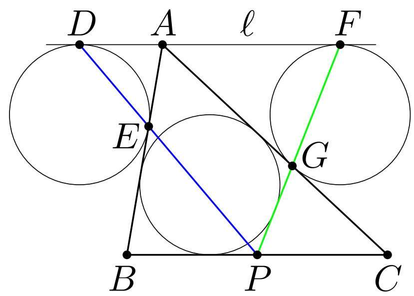
1. In triangle ABC, point O is the circumcenter. The line AO intersects BC at point T, and the perpendiculars drawn from T to AB and AC intersect the radii OB and OC at points E and F, respectively. Prove that $BE = CF$.

2. Given a triangle ABC, with a marked point I as its incenter, and K_1 and K_2 being the points of tangency of the incircle with sides BC and AC, respectively. Using a compass and a ruler, construct the incenter of triangle CK_1K_2 with the minimal possible number of lines (a line is a straight line or a circle).

3. Let ABC be a right triangle ($\angle C = 90°$), N be the midpoint of arc BAC of the circumcircle, and K the intersection point of CN with AB. On the extension of AK beyond K, let T be the point chosen such that $TK = KA$. Prove that the circle with center T and radius TK is tangent to BC.

4. Let ABC be an acute triangle, AD, BE, and CF its altitudes, and H the orthocenter. On the rays AD, BE, and CF, points A_1, B_1, and C_1 chosen such that $AA_1 = HD$, $BB_1 = HE$, and $CC_1 = HF$ respectively. Let A_2, B_2, and C_2 be the midpoints of A_1D, B_1E, and C_1F, respectively. Prove that the points H, A_2, B_2, and C_2 lie on the same circle.

5. Through vertex A of triangle ABC, a line $\ell \parallel BC$ is drawn. Two circles, each congruent to the incircle of triangle ABC, are tangent to the lines ℓ, AB, and AC as shown in the diagram. The lines DE and FG intersect at point P, which lies on BC. Prove that P is the midpoint of BC.



6. In an isosceles triangle ABC with $\angle BAC = 108°$, the bisector of angle ABC intersects the circumcircle of the triangle at point D. Point E on segment BC is such that $AB = BE$. Prove that the perpendicular bisector of CD is tangent to the circumcircle of triangle ABE.

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9th Grade

1. In an acute triangle ABC, the altitudes BD and CE intersect at point H. A point F is chosen on side AC, such that $FH \perp CE$. Segment FE intersects the circumcircle of triangle CDE at point K. Prove that $HK \perp EF$.

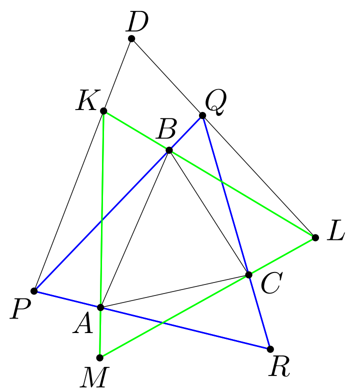
2. Let BC and BD be the tangents drawn from point B to the circle with diameter AC, and let E be the second intersection point of line CD with the circumcircle of triangle ABC. Prove that $CD = 2DE$.

3. Given a triangle ABC, with a marked point I as its incenter, and K_1 and K_2 being the points of tangency of the incircle with sides BC and AC, respectively. Using a compass and a ruler, construct the center of the excircle of triangle CK_1K_2 that is tangent to CK_2, using at most 4 lines (a line is a straight line or a circle).

4. Let BE and CF be the altitudes of an acute triangle ABC, H its orthocenter, M the midpoint of BC, K and L the intersection points of the perpendicular bisector of BC with BD and CE, respectively, and Q the orthocenter of triangle KLH. Prove that Q lies on the median AM.

5. Let I be the incenter of triangle ABC, and K the point of tangency of the incircle with side BC. Points X and Y are chosen on segments BI and CI, respectively, such that $KX \perp AB$ and $KY \perp AC$. The circumcircle of triangle XYK meets BC again at point D (other than point K). Prove that $AD \perp BC$.

6. Around an acute triangle ABC, equilateral triangles KLM and PQR are constructed as shown in the diagram. Lines PK and QL intersect at point D. Prove that $\angle ABC + \angle PDQ = 120°$.



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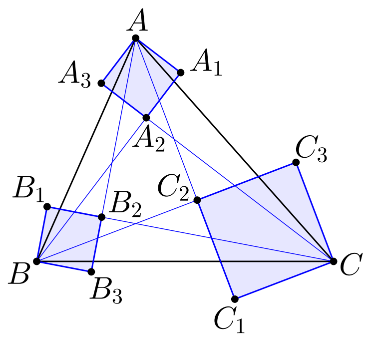
10-11th Grade

1. Circles ω_1 and ω_2 are tangent to a line ℓ at points A and B, respectively, and are tangent to each other externally at point D. A point E is chosen arbitrarily on the minor arc BD of circle ω_2. The line DE meets circle ω_1 at point C for the second time. Prove that $BE \perp AC$.

2. Let I be the incenter of triangle ABC, where $\angle A = 60°$, and let D be the point of tangency of the incircle with side BC. Points X and Y are chosen on segments BI and CI, respectively, such that $DX \perp AB$ and $DY \perp AC$. A point Z is chosen such that triangle XYZ is equilateral, and points Z and I lie on the same side of line XY. Prove that $AZ \perp BC$.

3. Given an acute triangle ABC. Squares $AA_1A_2A_3$, $BB_1B_2B_3$, and $CC_1C_2C_3$ are positioned such that the lines A_1A_2, B_1B_2, and C_1C_2 pass through points B, C, and A, respectively, and the lines A_2A_3, B_2B_3, and C_2C_3 pass through points C, A, and B, respectively. Prove that
a) the lines AA_2, B_1B_3, and C_1C_3 are concurrent;
b) the lines AA_2, BB_2, and CC_2 are concurrent.



4. On a semicircle with diameter AB, a point C is chosen arbitrarily. Let P and Q be points on segment AB such that $AP = AC$ and $BQ = BC$, and let O and H be the circumcenter and orthocenter of triangle CPQ, respectively. Prove that for all possible positions of point C, line OH passes through a fixed point.

5. Given a scalene triangle ABC, with the incenter I marked, and the points of tangency of the incircle with sides BC, AC, and AB marked as K_1, K_2, and K_3, respectively. Using only a ruler, construct the circumcenter of triangle ABC.

6. Given a scalene triangle ABC. Through point B, a line ℓ is drawn that does not intersect the triangle and forms distinct angles with sides AB and BC. Let M be the midpoint of AC, and let H_a and H_c be the feet of the perpendiculars drawn from points A and C to ℓ. The circumcircle of triangle MBH_a intersects AB at point A_1, and the circumcircle of triangle MBH_c intersects BC at point C_1. Point A_2 is symmetric to A with respect to point A_1, and point C_2 is symmetric to C with respect to point C_1. Prove that the lines ℓ, AC_2, and CA_2 are concurrent.

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