8th Grade

1. Let BE and CF be the medians of an acute triangle ABC. On the line BC, points $K \neq B$ and $L \neq C$ are chosen such that $BE = EK$ and $CF = FL$. Prove that $AK = AL$.

2. Let I be the incenter and O be the circumcenter of triangle ABC, where $\angle A < \angle B < \angle C$. Points P and Q are such that $AIOP$ and $BIOQ$ are isosceles trapezoids ($AI \parallel OP$, $BI \parallel OQ$). Prove that $CP = CQ$.

3. Let W be the midpoint of the arc BC of the circumcircle of triangle ABC, such that W and A lie on opposite sides of line BC. On sides AB and AC, points P and Q are chosen respectively so that $APWQ$ is a parallelogram, and on side BC, points K and L are chosen such that $BK = KW$ and $CL = LW$. Prove that the lines AW, KQ, and LP are concurrent.

4. On side AB of an isosceles trapezoid $ABCD$ ($AD \parallel BC$), points E and F are chosen such that a circle can be inscribed in quadrilateral $CDEF$. Prove that the circumcircles of triangles ADE and BCF are tangent to each other.

5. On side AC of triangle ABC, a point P is chosen such that $AP = \frac{1}{3}AC$, and on segment BP, a point S is chosen such that $CS \perp BP$. A point T is such that $BCST$ is a parallelogram. Prove that $AB = AT$.

December 22, 2024

9th Grade

1. Inside triangle ABC, a point D is chosen such that $\angle ADB = \angle ADC$. The rays BD and CD intersect the circumcircle of triangle ABC at points E and F, respectively. On segment EF, points K and L are chosen such that $\angle AKD = 180^\circ - \angle ACB$ and $\angle ALD = 180^\circ - \angle ABC$, with segments EL and FK not intersecting line AD. Prove that $AK = AL$.

2. Let M be the midpoint of side BC of triangle ABC, and let D be an arbitrary point on the arc BC of the circumcircle that does not contain A. Let N be the midpoint of AD. A circle passing through points A, N, and tangent to AB intersects side AC at point E. Prove that points C, D, E, and M are concyclic.

3. Let H be the orthocenter of an acute triangle ABC, and let AT be the diameter of the circumcircle of this triangle. Points X and Y are chosen on sides AC and AB, respectively, such that $TX = TY$ and $\angle XTY + \angle XAY = 90^\circ$. Prove that $\angle XHY = 90^\circ$.

4. Let ω be the circumcircle of triangle ABC, where $AB > AC$. Let N be the midpoint of arc $\smile BAC$, and D a point on the circle ω such that $ND \perp AB$. Let I be the incenter of triangle ABC. Reconstruct triangle ABC, given the marked points A, D, and I.

5. Let AL be the bisector of triangle ABC, O the center of its circumcircle, and D and E the midpoints of BL and CL, respectively. Points P and Q are chosen on segments AD and AE such that $APLQ$ is a parallelogram. Prove that $PQ \perp AO$.

December 22, 2024

10–11th Grade

1. Let I and O be the incenter and circumcenter of the right triangle ABC ($\angle C = 90°$), and let K be the tangency point of the incircle with AC. Let P and Q be the points where the circumcircle of triangle AOK intersects OC and the circumcircle of triangle ABC, respectively. Prove that points C, I, P, and Q are concyclic.

2. Let O and H be the circumcenter and orthocenter of the acute triangle ABC. On sides AC and AB, points D and E are chosen respectively such that segment DE passes through point O and $DE \parallel BC$. On side BC, points X and Y are chosen such that $BX = OD$ and $CY = OE$. Prove that $\angle XHY + 2\angle BAC = 180°$.

3. Inside triangle ABC, points D and E are chosen such that $\angle ABD = \angle CBE$ and $\angle ACD = \angle BCE$. Point F on side AB is such that $DF \parallel AC$, and point G on side AC is such that $EG \parallel AB$. Prove that $\angle BFG = \angle BDC$.

4. Let I and M be the incenter and the centroid of a scalene triangle ABC, respectively. A line passing through point I parallel to BC intersects AC and AB at points E and F, respectively. Reconstruct triangle ABC given only the marked points E, F, I, and M.

5. Let $ABCDEF$ be a cyclic hexagon such that $AD \parallel EF$. Points X and Y are marked on diagonals AE and DF, respectively, such that $CX = EX$ and $BY = FY$. Let O be the intersection point of AE and FD, P the intersection point of CX and BY, and Q the intersection point of BF and CE. Prove that points O, P, and Q are collinear.

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