IX Yasinskyi Geometry Olympiad

November 9, 2025

8th Grade[1]

1. Given a triangle ABC, a point D is chosen on the side BC, and a point E is chosen inside the triangle such that $\angle BAD = \angle ECD$ and $\angle DEC = \angle ABC$. Prove that $\angle BEC = 180^\circ - \angle BAC$.

2. On the side AC of triangle ABC, a point D is chosen such that $BD = CD$, and on the segment BD a point E is chosen such that $CE = AB$. Suppose that $AB + BE = AC$. Find $\angle BAC$.

3. Let M be the midpoint of the side BC of a triangle ABC, and let P and Q be the midpoints of the altitudes BE and CF respectively. Reconstruct the triangle ABC, given marked points M, P, and Q.

4. Let O be the circumcenter of an acute triangle ABC. Points D and E are chosen on the sides AB and AC respectively so that segment DE passes through point O. Let K and L be the orthocenters of triangles BOD and COE respectively, and let T be the intersection point of lines KD and LE. Prove that the points A, K, T, and L are concyclic.

5. Let ABC be an acute triangle with orthocenter H and circumcenter O. Suppose that there exists a point P on the side BC such that $OP = OH$ and $HP = AH$. Prove that the point P lies on line AO or on line AH.

Time allowed: 4 hours. Each problem is worth 7 points.

[1]The Olympiad problems are designed for students in the final four grades of a high school. If your school has a different from 11 number of grades, consider the problems for grades 10–11 to be intended for the last and second-to-last grades of your school, and so on.

IX Yasinskyi Geometry Olympiad

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9th Grade[1]

1. Let $ABCD$ be a cyclic quadrilateral. On the side AD, there exist points K and L such that $AK = BK$ and $CL = DL$, moreover points A, K, L, D lie on line AD in this order. Let M be a point such that $KM \parallel AB$ and $LM \parallel CD$. Prove that $BM = CM$.

2. In a triangle ABC, points P and Q are chosen on rays AC and BC respectively, so that the circumcircles of triangles ACQ and BCP are tangent to the line AB. Let O be the circumcenter of triangle PCQ. Prove that $AO = BO$.

3. Let BE and CF be the angle bisectors of triangle ABC. On the extension of line EF beyond F, a point P is chosen such that $AB = BP$, and on the extension of line FE beyond E, a point Q is chosen such that $AC = CQ$. Prove that $\angle BPQ = \angle CQP$.

4. Let $ABCD$ be a cyclic quadrilateral with $AD \nparallel BC$. Points X and Y are chosen on the sides AB and CD respectively such that $AX/XB = CY/YD$. Points P and Q are the reflections of the point X with respect to the lines AD and BC respectively. Prove that $PY = QY$.

5. Let O be the circumcenter of an isosceles triangle ABC $(AB = AC)$, K be the midpoint of the arc AB of the circumcircle which does not contain the point C, T be the point on line BO such that $\angle KAT = 90^\circ$, and E be the midpoint of AC. Prove that $\angle KET = 90^\circ$.

Time allowed: 4 hours. Each problem is worth 7 points.

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IX Yasinskyi Geometry Olympiad

November 9, 2025

10 – 11th Grade[1]

1. Let ABC be a triangle with $AB = AC = 2BC$. Denote ω its incircle and I its incenter. The circle ω is tangent to the side AC at a point K, and F is the second point of intersection of line BK with ω. Prove that the points A, I, F, and B are concyclic.

2. Let O be the circumcenter of an acute triangle ABC. On the sides AB and AC, points K and L are chosen respectively, such that $OK = BK$ and $OL = CL$. The circumcircles of triangles ABC and AKL meet again at a point T. Prove that $AT \parallel BC$.

3. Let $ABCD$ be a trapezoid $(AD \parallel BC)$. A point K is chosen on the side CD. Circles with centers I and J are inscribed in triangles BCK and ADK respectively. Find all trapezoids $ABCD$ for which it might happen that both polygons $ABIKJ$ and $DCIJ$ are cyclic.

4. Let ABC be a triangle with $AB \neq AC$. Points D, E, and F are chosen on the sides BC, AC, and AB respectively, such that quadrilateral $BFEC$ is cyclic, and the circumcircle of triangle DEF is tangent to BC at the point D. On line AD, there exists a point Q such that $BQ = CQ$, moreover the points A and Q are on different sides of the line BC. Prove that

$$\angle BAC + \angle EDF + \angle BQC = 180^\circ.$$

5. Inside an acute scalene triangle ABC, a point D is chosen such that $\angle ABD = \angle ACD$. Circle with diameter AD meets the circumcircle of the triangle ABC again at a point K and the altitude AH at a point E. Prove that line KE passes through the midpoint of the side BC.

Time allowed: 4 hours. Each problem is worth 7 points.

[1]The Olympiad problems are designed for students in the final four grades of a high school. If your school has a different from 11 number of grades, consider the problems for grades 10–11 to be intended for the last and second-to-last grades of your school, and so on.